

GPS Coordinate Estimation From Calibrated Cameras

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Abstract

We present a novel application where, using only three points from the shadow trajectory of an object, one can accurately determine the geo-location of the camera, up to a longitude ambiguity, and also the date of image acquisition without using any GPS or other special instruments. We refer to this as “geo-temporal localization”. We consider possible cases where ambiguities can be removed if additional information is available. Our method does not require any knowledge of the date or the time when the pictures are taken, and geo-temporal information is recovered directly from the images. We demonstrate the accuracy of our geo-temporal localization method using synthetic and real data.

1 Introduction

In ICCV 2005 a contest was run on a collection of color images acquired by an already calibrated digital camera. The photographs were taken at various locations and often shared overlapping fields of view, or had certain objects in common. More importantly, the GPS locations for a subset of these images were provided in advance. The goal of the contest was to guess, as accurately as possible, the GPS locations of the unlabeled images. This paper pushes the limits in the state of the art beyond what is currently known to be feasible from images in terms of geo-temporal localization solely based on computer vision techniques.

The cue that we use for *geo-temporal* localization of the camera, (defined henceforth as *the physical location of the camera (GPS coordinates) and the date of image acquisition*) is the shadow trajectories of two stationary objects during the course of a day. The use of shadow trajectory of a gnomon to measure time in a sundial is reported as early as 1500 BC by Egyptians, which surprisingly requires sophisticated astronomical knowl-

edge [8, 9, 13]. Shadows have been used in multiple-view geometry in the past to provide information about the shape and the 3-D structure of the scene [2, 5], or to recover camera intrinsic and extrinsic parameters [1, 4]. Determining the GPS coordinates and the date of the year from shadows in images is a new concept that we introduce in this paper.

Many approaches can be used to calibrate cameras by just observing shadows [3, 12] only. For geo-temporal localization, recently Jacobs et al. [10] use a database of images collected over a course of a year to learn weather patterns. Using these natural variations, camera is then geo-located by the correlation of camera images to geo-registered satellite images and also by correlating acquired images with known landmarks/locations. In contrast, the present work is based solely on astronomical geometry and is more flexible, requiring only three shadow points for GPS coordinate estimation. To demonstrate the power of the proposed method we downloaded some images from online traffic surveillance webcams, and determined accurately the geo-locations and the date of acquisition.

Observing at least two objects that cast shadows on the ground plane, we present an innovative application to estimate the GPS coordinates of the location where the images were taken, along with the day of year when the images were taken (up to year ambiguity). Only three points on the shadow trajectories are required, leading to a robust geo-temporal localization. Accordingly, this paper is divided into corresponding sections addressing each issue.

2 Preliminaries and the setup

Camera intrinsic parameters are given by the matrix

$$\mathbf{K} = \begin{bmatrix} \lambda f & \gamma & u_0 \\ 0 & f & v_0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (1)$$

where \mathbf{K} is a nonsingular 3×3 upper triangular matrix known as the camera calibration matrix including five parameters, i.e. the focal length f , the skew γ , the aspect ratio λ and the principal point at (u_0, v_0) . Another

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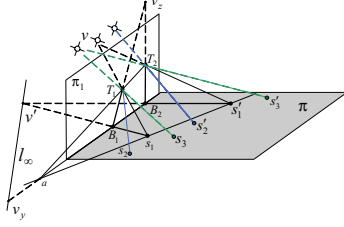


Figure 1. The setup used for camera calibration and for estimating geo-temporal information.

important quantity is the Image of the Absolute Conic (IAC), given as: $\omega = \mathbf{K}^{-T}\mathbf{K}^{-1}$. Once these intrinsic camera parameters are known, a camera can be treated as an angle measuring device [7, 11].

Let \mathbf{T} be a 3D stationary point and \mathbf{B} its footprint (i.e. its orthogonal projection) on the ground plane. As depicted in Fig. 1, the locus of shadow positions \mathbf{S} cast by \mathbf{T} on the ground plane, i.e. the shadow trajectory, is a smooth curve that depends only on the altitude (ϕ) and the azimuth angles (θ) of the sun in the sky and the vertical distance h of the object from its footprint. Thus, algebraically, the 3D coordinates of the shadow position can be unambiguously specified by their 2D coordinates in the ground plane as

$$\bar{\mathbf{S}}_i = \bar{\mathbf{B}}_i + h_i \cot \phi \begin{bmatrix} \cos \theta \\ \sin \theta \end{bmatrix}, \quad (2)$$

where $\bar{\mathbf{S}}_i = [S_{ix} \ S_{iy}]^T$ and $\bar{\mathbf{B}}_i = [B_{ix} \ B_{iy}]^T$ are the inhomogeneous coordinates of the shadow position \mathbf{S}_i , and the object's footprint \mathbf{B}_i on the ground plane. Equation (2) is based on the assumption that the sun is distant and therefore its rays, e.g. $\mathbf{T}_i\mathbf{S}_i$, are parallel to each other. Therefore, these parallel lines intersect at a point at infinity. This point then gets projected to a vanishing point \mathbf{v}' in the image plane. This point also lies on the horizon line, i.e. the line at infinity l_∞ , of the ground plane (cf. Figure 1).

3 The Geo-temporal Localization Step

At any time of the year, the exact location of the sun can be determined by two angles, i.e. azimuth and the altitude angle. Thus, when we have at least two vertical objects in the scene casting shadows on the ground plane, we can estimate these two angles. This is primarily due to the fact that vertical objects enable us to obtain the vertical vanishing point \mathbf{v}_z (cf. Figure 1). For this it is necessary that the world point casting the shadow on the ground plane be visible in the image.

The earth orbits the sun approximately every 365 days while it also rotates on its axis that extends from the north pole to the south pole every 24 hours. The orbit around the sun is elliptical in shape, which causes it to speed up and slow down as it moves around the sun. The polar axis also tilts up to a maximum angle of about 23.47° with the orbital plane over the course

of a year. This tilt causes a change in the angle that the sun makes with the equatorial plane, the so called *declination angle*. Similarly, the globe may be partitioned in several ways. A circle passing through both poles is called a *Meridian*. Another circle that is equidistance from the north and the south pole is called the *Equator*. *Longitude* is the angular distance measured from the prime meridian through Greenwich, England. Similarly, *Latitude* is the angular distance measured from the equator, North (+ve) or South (-ve). Latitude values are important as they define the relationship of a location with the sun. Also, the path of the sun, as seen from the earth, is unique for each latitude, which is the main cue which allows us to geo-locate the camera from only shadow trajectories. Next, we describe the methods for determining these quantities.

Latitude: An overview of the proposed method is shown in Fig. 1. Let \mathbf{s}_i , $i = 1, 2, 3$ be the images of the shadow points of a stationary object recorded at different times during the course of a single day. Let \mathbf{v}_i and \mathbf{v}'_i , $i = 1, 2, 3$ be the sun and the shadow vanishing points, respectively. For a calibrated camera, the following relations hold for the altitude angle ϕ_i and the azimuth angle θ_i of the sun orientations in the sky, all of which are measured directly in the image domain (cf. Figure 1)

$$\cos \phi_i = \frac{\mathbf{v}'_i{}^T \omega \mathbf{v}_i}{\sqrt{\mathbf{v}'_i{}^T \omega \mathbf{v}'_i} \sqrt{\mathbf{v}_i{}^T \omega \mathbf{v}_i}} \quad (3)$$

$$\sin \phi_i = \frac{\mathbf{v}_z{}^T \omega \mathbf{v}_i}{\sqrt{\mathbf{v}_z{}^T \omega \mathbf{v}_z} \sqrt{\mathbf{v}_i{}^T \omega \mathbf{v}_i}} \quad (4)$$

$$\cos \theta_i = \frac{\mathbf{v}_y{}^T \omega \mathbf{v}'_i}{\sqrt{\mathbf{v}_y{}^T \omega \mathbf{v}_y} \sqrt{\mathbf{v}'_i{}^T \omega \mathbf{v}'_i}} \quad (5)$$

$$\sin \theta_i = \frac{\mathbf{v}_x{}^T \omega \mathbf{v}'_i}{\sqrt{\mathbf{v}_x{}^T \omega \mathbf{v}_x} \sqrt{\mathbf{v}'_i{}^T \omega \mathbf{v}'_i}} \quad (6)$$

Without loss of generality, we choose an arbitrary point on the horizon line as the vanishing point \mathbf{v}_x along the x-axis, and the image point \mathbf{b} of the footprint/bottom as the image of the world origin. The vanishing point \mathbf{v}_y along the y-axis is then given by $\mathbf{v}_y \sim \omega \mathbf{v}_x \times \omega \mathbf{v}_z$.

Let ψ_i be the angles measured clockwise that the shadow points make with the positive x-axis as shown in Fig. 1. We have

$$\cos \psi_i = \frac{\mathbf{v}'_i{}^T \omega \mathbf{v}_x}{\sqrt{\mathbf{v}'_i{}^T \omega \mathbf{v}'_i} \sqrt{\mathbf{v}_x{}^T \omega \mathbf{v}_x}} \quad (7)$$

$$\sin \psi_i = \frac{\mathbf{v}'_i{}^T \omega \mathbf{v}_y}{\sqrt{\mathbf{v}'_i{}^T \omega \mathbf{v}'_i} \sqrt{\mathbf{v}_y{}^T \omega \mathbf{v}_y}} \quad i = 1, 2, 3 \quad (8)$$

Next, we define the following ratios, which are readily derived from spherical coordinates, and also used in

sundial construction:

$$\rho_1 = \frac{\cos \phi_2 \cos \psi_2 - \cos \phi_1 \cos \psi_1}{\sin \phi_2 - \sin \phi_1} \quad (9)$$

$$\rho_2 = \frac{\cos \phi_2 \sin \psi_2 - \cos \phi_1 \sin \psi_1}{\sin \phi_2 - \sin \phi_1} \quad (10)$$

$$\rho_3 = \frac{\cos \phi_2 \cos \psi_2 - \cos \phi_3 \cos \psi_3}{\sin \phi_2 - \sin \phi_3} \quad (11)$$

$$\rho_4 = \frac{\cos \phi_2 \sin \psi_2 - \cos \phi_3 \sin \psi_3}{\sin \phi_2 - \sin \phi_3} \quad (12)$$

For our problem, it is clear from (3)-(8) that these ratios are all determined directly in terms of image quantities. This is possible only because the camera has been calibrated. The angle measured at world origin between the positive y-axis and the ground plane's primary meridian (i.e. the north direction) is then given by

$$\alpha = \tan^{-1} \left(\frac{\rho_1 - \rho_3}{\rho_4 - \rho_2} \right) \quad (13)$$

from which we can determine the GPS latitude of the location where the pictures are taken as

$$\lambda = \tan^{-1}(\rho_1 \cos \alpha + \rho_2 \sin \alpha) \quad (14)$$

For n shadow points, we obtain a total of $\frac{n!}{(n-3)!3!}$ estimations of latitude(λ). In the presence of noise, this leads to a very robust estimation of λ .

Day Number: Once the latitude is determined from (14), we can also determine the exact day when the images are taken. For this purpose, let δ denote the declination angle (positive in the summer). Let also h denote the hour angle for a given image, i.e. the angle the earth needs to rotate to bring the meridian of that location to solar noon, where each hour time corresponds to $\frac{\pi}{12}$ radians, and the solar noon is when the sun is due south with maximum altitude. Then these angles are given in terms of the latitude λ , the sun's altitude ϕ and its azimuth θ by

$$\sin h \cos \delta - \cos \phi \sin \theta = 0 \quad (15)$$

$$\cos \delta \cos \lambda \cos h + \sin \delta \sin \lambda - \sin \phi = 0 \quad (16)$$

Again, note that the above system of equations depend only on image quantities defined in (3)-(8). Upon finding the declination and the hour angles by solving the above equations, the exact day of the year when the pictures are taken can be found by

$$N = \frac{365}{2\pi} \sin^{-1} \left(\frac{\delta}{\delta_m} \right) - N_o \quad (17)$$

where N is the day number of the date, with January 1st taken as $N = 1$, and February assumed of 28 days, $\delta_m \simeq 0.408$ is the maximum absolute declination angle of earth in radians, and $N_o = 284$ corresponds to the number of days from the first equinox to January 1st.

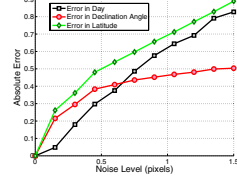


Figure 2. Result for average error in latitude, solar declination angle, and day of the year.

Longitude: The longitude may be determined by temporal correlation. For instance, suppose we have a few frames from a video stream of a live webcam with unknown location. Then they can be temporally correlated with our local time, in which case the difference in hour angles can be used to determine the longitude.

For this purpose, let h_l and γ_l be our own local hour angle and longitude at the time of receiving the live pictures. Then the GPS longitude of the location where the pictures are taken is given by

$$\gamma = \gamma_l + (h - h_l) \quad (18)$$

Therefore, by using only three shadow points, we are able to determine the geo-location up to longitude ambiguity, and specify the day of the year when the images were taken up to, of course, year ambiguity from using only images. The key observation that allows us to achieve this is the fact that a calibrated camera performs as a direction tensor, capable of measuring direction of rays and hence angles, and that the latitude and the day of the year are determined simply by measuring angles in images.

4 Experimental Results

Synthetic Data: Two vertical objects of different heights were randomly placed on the ground plane. Using the online available version of SunAngle Software [6], we generated altitude and azimuth angles for the sun corresponding to our own geo-location with latitude 28.51° . The data was generated for the 315th day of the year i.e. the 11th of November 2006 from 10:00am to 2:00pm. The solar declination angle for that time period is -17.49° . The vertical objects and the shadow points were projected by a synthetic camera with a focal length of $f = 1000$, the principal point at $(u_o, v_o) = (320, 240)$, unit aspect ratio, and zero skew.

To test for noise resilience, we gradually added Gaussian noise of zero mean and standard deviation of up to 1.5 pixels to the projected points. Averaged results for latitude, solar declination angle, and the day of the year are shown in Figure 2. The error is found to be less than 0.9%. For a maximum noise level of 1.5 pixels, the estimated latitude is 28.21° , the declination angle is -17.932° , and the day of the year is found to be 314.52.

Table 1. Results for 11 sets of 10-image combination. Mean value and standard deviation for latitude is found to be $(38.743^\circ, 3.57)$, $(-16.43^\circ, 1.11)$ for the declination angle, and $(329.95, 2.28)$ for the estimated number of the day.

| | C_1 | C_2 | C_3 | C_4 | C_5 | C_6 | C_7 | C_8 | C_9 | C_{10} | C_{11} |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|----------|----------|
| Lat. | 33.73 | 35.70 | 37.03 | 36.1 | 35.72 | 38.21 | 39.23 | 45.78 | 41.84 | 40.88 | 41.96 |
| Dec. | -14.47 | -15.78 | -15.93 | -16.54 | -17.25 | -16 | -16.70 | -18.94 | -15.87 | -16.99 | -16.24 |
| Day # | 328.64 | 332.26 | 331.09 | 326.87 | 330.15 | 331.37 | 331.32 | 332.56 | 326.81 | 331.72 | 326.72 |

Real Data: Experiments on real data sets are reported below for demonstrating the power of the proposed method. 11 images were captured live from downtown Washington D.C. area, using one of the webcams available online at <http://trafficland.com/>. As shown in Figure 3, a lamp post and a traffic light were used as two objects casting shadows on the road. The shadow points are highlighted by colored circles in the figure. The calibration parameters were estimated as

Since we had more than the required minimum number of shadow locations over time, in order to make the estimation more robust to noise, we took all possible combinations of the available points and averaged the results. For this first data set the images were captured on the 15th November at latitude 38.53° and longitude 77.02° . We estimated the latitude as 38.74° , the day number as 329.95 and the solar declination angle as -16.43° compared to the actual day of 319, and the declination angle of -18.62° . The small errors can be attributed to many factors e.g. noise, non-linear distortions and errors in the extracted features in low-resolution images of 320×240 . Despite all these factors, the experiment indicates that the proposed method provides good results.

In order to evaluate the uncertainty associated with our estimation, we then divided this data set into 11 sets of 10-image combinations, i.e. in each combination we left one image out. We repeated the experiment for each combination and calculated the mean and the standard deviation of the estimated unknown parameters. Results are shown in Table 1. The low standard deviations can be interpreted as small uncertainty, indicating that our method is consistently providing reliable results.

5 Conclusion

We propose a method based entirely on computer vision to determine the geo-location of the camera up to longitude ambiguity, without using any GPS or other instruments, and by solely relying on imaged shadows as cues. We also describe situations where longitude ambiguity can be removed by either temporal or spatial cross-correlation. Moreover, we determine the date when the pictures are taken without using any prior information. Good results indicate the practicality of the proposed approach.



Figure 3. Few of the images taken from one of the live webcams in downtown Washington D.C. The two objects that cast shadows on the ground are shown in red and blue, respectively. Shadows move to the left of the images as time progresses.

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