BLIND ESTIMATION OF PSF FOR OUT OF FOCUS VIDEO DATA

H. Shekarforoush *R. Chellappa*

Center For Automation Research
University of Maryland
College Park, MD 20742, U.S.A.
email: {hshekar,rama}@cfar.umd.edu

ABSTRACT

A method has been proposed for blindly estimating the Point Spread Function (PSF) of video data. The PSF's of the images in a sequence are assumed to be of compact support and hence admit FIR modeling. The zeros (roots) of the Optical Transfer Function (OTF) - i.e. the Fourier transform of the PSF - are first estimated by locating the singularities of the magnitude of the cross power spectrum. These roots are then used for spectrum factorization leading to complex polynomial approximation of the OTF. It has been shown that a 2D spectrum can be factorized under the assumption of local isotropy around the roots.

1. INTRODUCTION

Estimating the PSF blindly is a challenging problem undertaken by numerous researchers in recent years. To overcome lack of sufficient information (i.e. blindness), a priori information need to be introduced. The following approaches can be identified in the literature:

1. Parametric model fitting approach [4][7][8]
2. Optimization-based approach [1][14][15][21].
3. Adaptive estimation [12][13][18]
4. Using higher order statistics [2][10][11]

All these methods, in fact, introduce the a priori information by imposing empirical models either on the actual PSF or the statistics of the PSF and the input signal. The use of higher order statistics in recent years has proven to be a very efficient approach, but at the cost of increased complexity and non-Gaussianity assumption of the statistics of the input signal. The approach adopted in this work is based on using cross-statistics rather than auto-statistics, and hence increased efficiency is achieved without increased complexity. Clearly the method would then require at least two observations of the same signal.

The proposed method is based on identifying the zeros of the OTF and factorizing the spectrum. The idea of characterizing the PSF by finding and using the zeros of the OTF is, in fact, relatively old [8]. The approach, however, has been abandoned due to the difficulty of detecting the zeros of the OTF in noise. Use of a denoising stage proposed in later literature [5][7] does not improve the estimation of the zeros either, as was originally reported in [3], too. Herein, we will demonstrate that the zeros can be readily identified using cross-statistics rather than the spectrum of the observed signal itself. Moreover, it will be shown that the spectrum factorization in 2D has an asymptotic solution although no direct solution is known. The following section will describe the method followed by a section in designing a simple stable inverse filter for testing the method. Results are illustrated in the final section.

This work was partially supported by the ONR/MURI grant N00014/95/1/0521 (ARPA order C635)

2. MODELING THE PSF

In this section, we will define our model for the PSF (or the OTF). We will then develop a method for the blind estimation of the OTF. As we will see in the next section, the proposed method is particularly applicable to video data, since it requires two images with only relative shifts between them. In fact, in video sequences transformations between successive frames can be closely approximated by shifts and a small rotation angle which can be compensated by registration.

We will show below how an adaptive PSF model can be built assuming local isotropy. For this purpose we will assume that the OTF of the imaging system is linear shift invariant and of finite duration and hence can be represented by a FIR filter:

$$\hat{h}(\mathbf{u}) = \sum_{\mathbf{k}\in\mathbf{T}} c_k exp(-i\langle\mathbf{u},\mathbf{k}\rangle) \tag{1}$$

where c_k's are some constants, $\langle,\rangle$ is the usual inner product in $\mathbb{R}^2$, and $\mathbf{T}$ is a discrete compact subset of $\mathbb{R}^2$.

As is well known [16], due to the fundamental theorem of algebra, in the univariate case the transfer function $\hat{h}$ can be factorized as the product of its roots. Clearly $\hat{h}$ is then specified up to a scale factor, if its roots are known. Unfortunately, due to general inability of factorizing polynomials in higher dimensions, this simple convenient factorization of the spectrum in univariate problems does not extend to higher dimensions. However, we will show that for the 2D case the problem can have a solution in an asymptotic sense. We start by assuming that the zeros of our OTF occur at isolated points[1]. Since the OTF is then either locally convex

[1] In general, the zeros of an entire function of two or more variables do not have to be isolated. However, in practice they can typically occur at isolated points [19].

0-8186-8821-1/98 $10.00 © 1998 IEEE

or concave around its zeros, one may assume local isotropy around a small neighborhood of the roots. The following results will show how a two dimensional spectrum with isolated zeros can be factorized.

Proposition 2.1 *Let $\{\zeta_n\}_{n\in Z}$ be a set of isolated points in $\mathbb{C}^n$. Then any function defined over $\mathbb{C}^n$, which is locally isotropic around these points, can be decomposed into a sum of functions, each globally isotropic around only one distinct point in the set.*

The existence of such a decomposition under appropriate boundary conditions is obvious. However, the decomposition is not necessarily unique, suggesting that the inverse may not necessarily be true. In fact counter examples can be readily found.

Theorem 2.1 *Let $\hat{h}_k(u,v)$ be the spectrum of a band-limited function vanishing outside a compact support $\mathbf{T} \subset \mathbb{R}^2$. Let also $\hat{h}_k(u,v)$ be isotropic around the point (u_k, v_k). Then:*

$$\lim_{b_k \to 0} b_k \hat{h}_k(u,v) = exp(-i\omega t_k) \qquad (2)$$

where $\omega = ((u-u_k)^2 + (v-v_k)^2)^{\frac{1}{2}}$ and $t_k = (x_k^2 + y_k^2)^{\frac{1}{2}}$ with $(x_k, y_k) \in \mathbf{T}$.

See the Appendix for proof.

Let us, now, assume that the OTF is locally isotropic around its roots then according to Proposition 2.1 we can write:

$$\hat{h}(u,v) = \sum_k \hat{h}_k(u,v) \qquad (3)$$

where each $\hat{h}_k$ is globally isotropic around one distinct root of $\hat{h}$. We shall denote the roots of $\hat{h}$ by the set $\{(u_k, v_k)\}_{k\in K}$, where K is a compact index set.

Therefore, under local isotropy assumption:

$$\hat{h}(u,v) = \sum_k a_k exp(-i\omega t_k) \quad where \ a_k = b_k^{-1} \qquad (4)$$

To physically interpret this convergence at the limit, note that b_k's define the radii of local isotropy around the roots of the OTF. Therefore the more local is the isotropy the better the OTF can be approximated by an expansion of the form in (4). This assumption of local isotropy, clearly relaxes the severe constraints usually imposed in the literature by global symmetries.

2.1. Spectral Factorization

In classical literature this term is used when it is required to find a function whose power spectrum is known. The problem has a solution if the power spectrum satisfies the Paley-Wiener condition [17]. Below, we will use the term in a more general context where two functions are sought when their cross power spectrum is known. We will show that the solution can be found if the functions are of compact support (e.g. FIR filters). Support constraint condition is clearly equivalent to Paley-Wiener condition due to the uncertainty principle [17].

From the expansion in (4) and the fundamental theorem of algebra, it immediately follows that:

$$\hat{h}(u,v) = A \prod_k (1 - c_k z^{-1}) \qquad (5)$$

where A and c_k's are some constants and $z = exp(i\omega) = exp\left(i\left((u-u_k)^2 + (v-v_k)^2\right)^{\frac{1}{2}}\right)$.

Clearly, the factorization is only valid if c_k's are identically equal to unity. Therefore, if the roots of the OTF are specified, then the OTF is known up to a scale factor. We will deal with the scale factor shortly. Let us first see how the roots of the OTF can be estimated from two observations of the same scene.

Consider the case where an object function has been observed by two systems modeled as follows:

$$\hat{g}_1 = \hat{h}_1 \hat{f}_1 \qquad (6)$$

$$\hat{g}_2 = \hat{h}_2 \hat{f}_2 \qquad (7)$$

where f_2 is a shifted version of f_1, $\hat{h}_1$ and $\hat{h}_2$ are the the OTF's and $\hat{g}_1$ and $\hat{g}_2$ are the observed image spectra[2].

We can now notice that the magnitude of the cross power spectrum will be given by:

$$|\ \hat{g}_1 \hat{g}_2^* \ | = c \ |\ \hat{h}_1 \hat{h}_2^* \ | \quad \text{where } c = |\hat{f}_1| = |\hat{f}_2| \qquad (8)$$

Therefore, since both $\hat{h}_1$ and $\hat{h}_2$ are assumed to be FIR filters, the poles of the cross power spectrum will be uniquely determined by the roots of $\hat{h}_2$. In practice this implies that the roots of $\hat{h}_2$ will be present in the form of some singularities appearing as a set of spikes scaled at different frequencies. In other words the roots can be identified by simple inspection of the magnitude of the cross power spectrum. Figure 1 shows an example of these spike patterns.

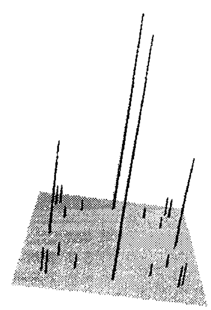


Figure 1: Typical spike patterns appearing in the magnitude of a cross power spectrum corresponding to isolated zeros of one of the OTF's

[2]When $\hat{f}_1$ and $\hat{f}_2$ are uniformly bounded away from zero on their support, then by replacing $\hat{h}_1$ by $\hat{h}_1 + \hat{n}_1 \hat{f}_1^*$ and $\hat{h}_2$ by $\hat{h}_2 + \hat{n}_2 \hat{f}_2^*$ (* denoting the conjugate) one can include a noise process in the model, too.

To determine the scale factor A, we will assume that the OTF preserves the mean value, and hence:

$$A = \frac{1}{\prod_k (1 - exp(-i\gamma_k))} \tag{9}$$

where $\gamma_k = (u_k^2 + v_k^2)^{\frac{1}{2}}$

Therefore, by identifying the singularities (spikes) in the magnitude of the cross power spectrum, we can completely specify the corresponding OTF.

3. A PSEUDO-INVERSE FILTER

Although the main issue in this work is the estimation and not the inversion of the PSF, we shall devise a simple method for constructing a pseudo-inverse filter φ. One may consider several approaches for stable pseudo-inversion, for instance by using an approximation around the origin. Using the biorthogonality condition ($\hat{h}\hat{\varphi} = 1$), we can derive the following identity:

$$(1 + \hat{\varphi})^{-1} \equiv \hat{h}(1 + \hat{h})^{-1} \tag{10}$$

A first order expansion of the left hand side around the origin will yield:

$$1 - \hat{\varphi} + \mathcal{O}(\hat{\varphi}^2) \simeq \hat{h}(1 + \hat{h})^{-1} \tag{11}$$

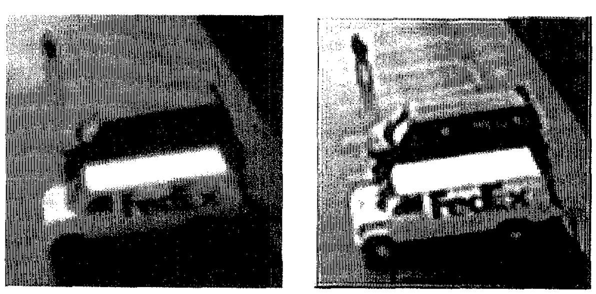
and hence:

$$\hat{\varphi} \simeq \frac{\hat{h}^*}{\hat{h}\hat{h}^* + \hat{h}^*} \tag{12}$$

which has some resemblance to the standard Wiener filter.

4. IMPLEMENTATION AND RESULTS

We have implemented the above method and applied it to successive frames in video data. Every two frames are first registered and rotation compensated using the methods proposed in [6] and [20]. Some results are shown below. Note that the result of applying the pseudo-inverse filter only partially reflects the performance of the PSF estimation, since there is a performance loss due to the fact that, only a pseudo-inverse filter could be constructed.



(a) (b)

Figure 2: (a) A blurred image, (b) its inversion

(a) (b)

Figure 3: (a) A blurred image, (b) its inversion

(a) (b)

Figure 4: (a) A blurred image, (b) its inversion

Appendix

Proof of theorem 2.1: First, a well known result is applied by the following change of variables to polar coordinates:

$$u - u_k = \omega cos(\varphi) \qquad v - v_k = \omega sin(\varphi)$$
$$x = tcos(\theta) \qquad y = tsin(\theta) \tag{13}$$

which leads to the following representation of the Fourier inversion theorem for $\hat{h}$:

$$h_k(t)exp(-it\rho_k) = \int_0^{\infty} \omega \hat{h}_k(\omega) J_0(t\omega) d\omega \tag{14}$$

where $\rho_k = u_k cos(\theta) + v_k sin(\theta)$, $\tilde{h}_k(\omega) = h_k(u, v)$ stands for the Hankel transform of $h_k(t) = h_k(x, y)$ and J_0 is the zero order Bessel function of the first kind.

Applying Hankel inversion theorem we then find:

$$\tilde{h}_k(\omega) = \int_0^{b_k} t h_k(t) exp(-it\rho_k) J_0(t\omega) dt \tag{15}$$

where b_k is the radius of local isotropy.

Since at $(u,v) = (u_k, v_k)$ we have $\tilde{h}_k(\omega) = 0$, we can write:

$$\int_0^{b_k} t h_k(t) exp(it\rho_k) dt = 0 \quad (16)$$

Substituting from (14) into this last integral equation, we will get:

$$\int_0^{b_k} t \int_0^{\infty} \omega \tilde{h}_k(\omega) J_0(t\omega) d\omega dt = 0 \quad (17)$$

which after changing the order of integration will yield:

$$\int_0^{\infty} \omega \tilde{h}_k(\omega) \int_0^{b_k} t J_0(t\omega) dt d\omega = 0 \quad (18)$$

Using the well known identity:

$$\int_0^{a} t J_0(t) dt = t J_1(at) \quad (19)$$

we deduce that:

$$\int_0^{b_k} t J_0(t\omega) dt = \frac{b_k J_1(b_k\omega)}{\omega} \quad (20)$$

Therefore:

$$\int_0^{\infty} b_k \tilde{h}_k(\omega) J_1(b_k\omega) d\omega = 0 \quad (21)$$

Finally, using the following integral relation for Bessel functions of the first kind [9]:

$$\int_0^{\infty} J_n(b\omega) exp(\pm i\omega a) = \frac{i^{\pm(n+1)}}{\sqrt{a^2 - b^2}} \left(a - \sqrt{a^2 - b^2}\right)^n \quad (22)$$

we conclude that:

$$\lim_{b_k \to 0} b_k \hat{h}_k(\omega) = exp(-i\omega t_k) \quad (23)$$

where $t_k = (x_k^2 + y_k^2)^{\frac{1}{2}}$ and (x_k, y_k) is a constant vector in **T**.

References

[1] A.J. Bell and T.J. Sejnowski. An information-maximization approach to blind seperation and blind deconvolution. *Neural Computation*, 7:1129–1159, 1995.

[2] D.H. Brooks and C.L. Nikias. Multichannel adaptive blind deconvolution using the complex cepstrum of higher order cross-spectra. *IEEE Trans. Signal Proc.*, 41(9):2928–2934, 1993.

[3] M. Cannon. Blind deconvolution of spatially invariant image blurs with phase. *IEEE Trans. ASSP*, 24(1):58–63, 1976.

[4] B. Chalmond. PSF estimation for image deblurring. *CVGIP, Graphical Models and Image Processing*, 53(4):364–372, 1991.

[5] P. Chen, M.A. Fiddy, A.H. Greenaway, and Y. Wang. Zero estimation for blind deconvolution from noisy sampled data. In *Proc. SPIE, Digita Image Recovery and Synthesis II*, volume 2029, pages 16–24, 1993.

[6] E. De-Castro and C. Morandi. Registration of translated and rotated images using finite Fourier transforms. In *IEEE PAMI*, volume 9, No 5, pages 700–703, 1987.

[7] R. Fabian and D. Malah. Robust identification of motion and out-of-focus blur parameters from blurred and noisy images. *CVGIP, Graphical Models and Image Processing*, 53(5):403–412, 1991.

[8] D. Gennery. Determination of optical transfer function by inspection of frequency-domain plot. *JOSA*, 63(12):1571–1577, 1973.

[9] A. Gray and T. M. Mathews. *A Treatise on Bessel Functions and Their Applications to Physics.* Dover Publications Inc., 1966.

[10] D. Hatzinakos and C.L. Nikias. Blind equalization using a tricepstrum based algorithm. *IEEE Trans. Communications*, COM-39:669–682, 1991.

[11] D. Hatzinakos and C.L. Nikias. *Blind Equalization Based on Higher Order Dtatistics*, chapter 5 in Blind Deconvolution, ed. S. Haykin. Prentice Hall, 1994.

[12] J.J. Hsue and A.E. Yagle. Blind deconvolution of symmetric noncausal impulse responses using two-sided linear prediction. *IEEE Trans. Signal Proc.*, 42(6):1509–1518, 1994.

[13] C. Jutten and J. Herault. Blind seperation of sources, part i: An adaptive algorithm based on neuromimetic architecture. *Signal Proc.*, 24:1–10, 1991.

[14] A.M. Katsaggelos and K.T. Lay. Maximum likelyhood blur identification and image restoration using expection-maximization algorithm. *IEEE Trans. Sig. Proc.*, 39(3):729–733, 1991.

[15] R.L. Lagendijk, A.M. Tekalp, and J. Biemond. Maximum-likelyhood image and blur identification: a unifying approach. *Opt. Eng.*, 29(5):422–435, 1990.

[16] A. V. Oppenheim and R. W. Schafer. *Discrete-Time Signal Processing.* Prentice-Hall International Inc., 1989.

[17] A. Papoulis. *Signal Analysis.* McGraw-Hill, 1977.

[18] S.J. Reeves and R.M. Mersereau. Blur identification by the method of generalized cross validation. *IEEE Trans. Image Proc.*, 1(3):301–311, 1992.

[19] M. S. Scivier and M. A. Fiddy. Phase ambiguities and the zeros of multidimensional band-limited functions. *J. Opt. Soc. Am.*, 2(5):693–697, 1985.

[20] M. Unser, P. Thévenaz, and L. Yaroslavsky. Convolution-based interpolation for fast high-quality rotation of images. *IEEE Trans. Image Processing*, 4(10):1371–1381, 1995.

[21] D.G. Warrier and U.B. Desai. Point spread function estimation using the mean field approximation. In *Proc. ICASSP*, pages 2599–2602. IEEE, 1995.